Common Calculus Mistakes
Chain Rule: Natural Logarithm

Some problems provide the opportunity for more than one mistake.

The Goal

Find

$$\frac{d}{dx}(ax\ln(bx)) \text{ (where } a \text{ and } b \text{ are constants)}$$

The Mistakes

Find the mistakes:

1.

$$\frac{d}{dx}(ax\ln(bx)) = a(1)\cdot\frac{1}{bx}$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dx}(ax\ln(bx)) = a(1)\cdot\frac{1}{bx}$$

2.

$$\frac{d}{dx}(ax\ln(bx)) = a\ln(bx)+\frac{ax}{bx}$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dx}(ax\ln(bx)) = a\ln(bx)+\frac{ax}{bx}$$

3.

$$\frac{d}{dx}(ax\ln(bx)) = a\ln(bx)+ax\cdot\frac{1}{x}\cdot b$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dx}(ax\ln(bx)) = a\ln(bx)+ax\cdot\frac{1}{x}\cdot b$$

A Correct Solution

$$\frac{d}{dx}(ax\ln(bx)) = a\ln(bx) + ax\cdot\frac{1}{bx}\cdot b = a\ln(bx) + a$$

Explanations

In the first mistake the product rule is not used, and in addition the chain rule is not used to find the derivative of ln(bx). The same failure to use the chain rule occurs in the second mistake, while in the third mistake the chain rule is used incorrectly.

The *chain rule* is applied to the natural logarithm of a function g(x) as follows:

$$\frac{d}{dx}(f(g(x)) = f'(g(x))g'(x) \implies \frac{d}{dx}(\ln(g(x)) = \frac{1}{g(x)}g'(x)$$

In this example g(x) = bx, so g'(x) = b.