Common Calculus Mistakes
Compound Chain Rule

Some problems provide the opportunity for more than one mistake.

The Goal

Find

$$\frac{d}{dt}\left(e^{t^2}\sqrt{\cos(e^t)}\right)$$

The Mistakes

Find the mistakes:

1.

$$\frac{d}{dt}\left(e^{t^2}\sqrt{\cos(e^t)}\right) = e^{t^2}2t\sqrt{\cos(e^t)} + e^{t^2}\sqrt{\cos(e^t)}(-\sin(e^t))e^t$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dt}\left(e^{t^2}\sqrt{\cos(e^t)}\right) = e^{t^2}2t\sqrt{\cos(e^t)} + e^{t^2}\sqrt{\cos(e^t)}(-\sin(e^t))e^t$$

2.

$$\frac{d}{dt}\left(e^{t^2}\sqrt{\cos(e^t)}\right) = e^{t^2}2t\sqrt{\cos(e^t)} + e^{t^2}\frac{1}{2}(\cos(e^t))^{-\frac{1}{2}}e^t$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dt}\left(e^{t^2}\sqrt{\cos(e^t)}\right) = e^{t^2}2t\sqrt{\cos(e^t)} + e^{t^2}\frac{1}{2}(\cos(e^t))^{-\frac{1}{2}}\cdot ?\cdot e^t$$

A Correct Solution

$$\frac{d}{dt}\left(e^{t^2}\sqrt{\cos(e^t)}\right) = e^{t^2}2t\sqrt{\cos(e^t)} + e^{t^2}\frac{1}{2}(\cos(e^t))^{-\frac{1}{2}}(-\sin(e^t))e^t$$

Explanations

In this example when the chain rule is used to differentiate the square root expression, it must be used twice in succession. That's because this expression is the composite of a composite. In the both mistakes this compound chain rule computation is not performed correctly. The chain rule says that the derivative of f(g(h(t))) is:

$$\frac{d}{dt}f(g(h(t))) = f'(g(h(t))) \cdot g'(h(t)) \cdot h'(t)$$

In this example:

$$f(t) = \sqrt{t}, g(t) = \cos(t) \text{ and } h(t) = e^t$$

Here's an algebra challenge: simplify the answer given in "A Correct Solution". One result is:

$$\frac{d}{dt}\left(e^{t^2}\sqrt{\cos(e^t)}\right) = \frac{(4\cos(e^t) - \sin(e^t))e^{t^2+t}}{2\sqrt{\cos(e^t)}}$$

<-- Back