Common Calculus Mistakes

Example: l'Hôpital's Rule

Some problems provide the opportunity for more than one mistake.

The Goal

Determine

$$\lim_{x\to 0^+}(ax\ln(bx))$$

The Mistakes

Find the mistakes:

1.

$$\lim_{x\to 0^+}(ax\ln(bx)) \overset{\text{l'H}}{=} \lim_{x\to 0^+}\left(a\ln(bx)+ax\cdot\frac{1}{bx}\cdot b\right)=\ldots$$

Need a hint? Look carefully at the red part:

$$\lim_{x\to 0^+}(ax\ln(bx)) \overset{\text{l'H}}{=} \lim_{x\to 0^+}\color{red}{\left(a\ln(bx)+ax\cdot\frac{1}{bx}\cdot b\right)}=\ldots$$

(The notation "l'H" above the equals sign indicates a step at which l'Hôpital's rule is claimed to be used)

2.

$$\lim_{x\to 0^+}(ax\ln(bx)) = \lim_{x\to 0^+}\frac{\ln(bx)}{\frac{1}{ax}} \overset{\text{l'H}}{=} \lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{1}{ax^2}} = -\frac{1}{a}$$

Need a hint? Look carefully at the red part:

$$\lim_{x\to 0^+}(ax\ln(bx)) = \lim_{x\to 0^+}\frac{\ln(bx)}{\frac{1}{ax}} \overset{\text{l'H}}{=} \lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{1}{ax^2}} = \color{red}{-\frac{1}{a}}$$

A Correct Solution

$$\lim_{x\to 0^+}(ax\ln(bx)) = \lim_{x\to 0^+}\frac{\ln(bx)}{\frac{1}{ax}} \overset{\text{l'H}}{=} \lim_{x\to 0^+}\frac{\frac{1}{x}}{-\frac{1}{ax^2}} = \color{blue}{\lim_{x\to 0^+}\frac{1}{x}\cdot(-ax^2)} = \lim_{x\to 0^+}(-ax) = 0$$

Explanations

In the first mistake the student has misunderstood l'Hôpital's rule to apply to a *product*. L'Hôpital's rule can be used to find the limit of a *quotient* when both the numerator and denominator are differentiable on an open interval containing the point of interest (or on an infinite open interval if the limit is to be taken at ±∞) *and* when the limit has indeterminate form, that is, the numerator and denominator both approach zero in the limit or each approaches either ∞ or -∞. Thus the first step is to rearrange the limit expression into the form of an indeterminate quotient.

The second mistake shows one way to correctly rearrange the expression; the mistake comes only at the end of the calculation when the final rational expression is simplified incorrectly.