
Common Algebra Mistakes

Function Notation

Some problems provide the opportunity for more than one mistake.

The Goal

Evaluate $f(x+h)$ for the function:

$$f(x) = \sqrt{1-x^2}$$

The Mistakes

Find the mistakes:

1.

$$f(x+h) = \sqrt{1-x^2} + h$$

Need a hint? Look carefully at the red part of the algebra:

$$f(x+h) = \sqrt{1-x^2} + h$$

2.

$$f(x+h) = \sqrt{1-x^2+h}$$

Need a hint? Look carefully at the red part of the algebra:

$$f(x+h) = \sqrt{1-x^2+h}$$

A Correct Solution

$$f(x+h) = \sqrt{1-(x+h)^2}$$

Explanations

In the first mistake $f(x+h)$ is computed to be $f(x)+h$. There are *very few* functions $f(x)$ for which the equation $f(x+h) = f(x) + h$ is true (it is true if $y = f(x)$ is a straight line with slope 1). The mistake in thinking that $f(x+h) = f(x) + h$ is true in general comes from not properly understanding function notation.

In $f(x)$ the x is a *placeholder*. To evaluate $f(x+h)$ we must put $x+h$ in for that placeholder *precisely where x appears* in $f(x)$ - no more and no less.

In the second example the $+h$ is again in the wrong place.

Follow the sequence of examples:

The function $f(x)$:

$$f(x) = \sqrt{1 - x^2}$$

Whatever we replace x with replaces x in the formula (even something silly like #):

$$f(\#) = \sqrt{1 - \#^2}$$

Replace x with 0.5 to find $f(0.5)$:

$$f(0.5) = \sqrt{1 - 0.5^2}$$

Note that 0.5 can be written as $0.4 + 0.1$:

$$f(0.4 + 0.1) = \sqrt{1 - (0.4 + 0.1)^2}$$

Following the pattern we can find $f(0.4+h)$:

$$f(0.4 + h) = \sqrt{1 - (0.4 + h)^2}$$

And $f(x+h)$ is a matter of replacing the 0.4 by x :

$$f(x + h) = \sqrt{1 - (x + h)^2}$$