## Common Algebra Mistakes Example: Solving an Exponential Equation

## The Goal

Solve the following equation for t:

$$2000e^{0.06t} = 4000$$

## The Mistake

Find the algebra mistake:

 $2000e^{0.06t} = 4000 \implies \ln t \cdot 2000e^{0.06} = \ln 4000 \implies e^{0.06t} = 0.5$  $\implies 0.06t = \ln 0.5 \implies t = \frac{\ln 0.5}{0.06}$ 

Need a hint? Look carefully at the red part of the algebra:

$$2000e^{0.06t} = 4000 \implies \ln t \cdot 2000e^{0.06} = \ln 4000 \implies e^{0.06t} = 0.5$$
$$\implies 0.06t = \ln 0.5 \implies t = \frac{\ln 0.5}{0.06}$$

The Correction

$$2000e^{0.06t} = 4000 \implies \ln 2000 + t \ln(e^{0.06}) = \ln 4000$$
$$\implies 0.06t = \ln 4000 - \ln 2000 \implies 0.06t = \ln \frac{4000}{2000} = \ln 2 \implies t = \frac{\ln 2}{0.06t}$$

## An Explanation

The rules of logarithms were used incorrectly in the first step - the t cannot be pulled in front of the 2000 since 2000 was not raised to the power t. "The Correction" shows how the rules could have been applied. The second step in "The Mistake" is a mystery; perhaps the first step was ignored and from the initial equation the student divided incorrectly. *Take care with the simple things.* 

For this equation a more efficient solution would be to start by dividing both sides by 2:

$$2000e^{0.06t} = 4000 \implies e^{0.06t} = \frac{4000}{2000} = 2 \implies 0.06t = \ln 2 \implies t = \frac{\ln 2}{0.06}$$

Notice how this solution isolates t one step at a time, from the outside to the inside. First

divide by 2000. This let's us undo the exponential by taking the natural logarithm of both sides. Finally, divide by 0.06, the remaining coefficient of t.