Common Trigonometry Mistakes

Example: Solve a trigonometric equation

Some problems provide the opportunity for more than one mistake.

The Goal

Solve the equation:

$$3\cos(2x) + 2\sin^2(x) = 0$$

The Mistakes

Find the mistakes:

1.

$$3\cos(2x) + 2\sin^2(x) = 0 \implies 3 \cdot 2\cos^2(x) - 1 + 2\sin^2(x) = 0$$

$$\implies 6\cos^2(x) - 1 + 2\sin^2(x) = 0 \implies 2(3\cos^2 - 1 + \sin^2(x)) = 0$$

$$\implies 3\cos^2(x) - 1 + 1 - \cos^2(x) = 0 \implies 2\cos^2(x) = 0 \implies \cos^2(x) = 0\cos(x) = 0$$

$$\implies x = \frac{\pi}{2} + k\pi \text{ (where } k \text{ is an integer)}$$

Need a hint? Look carefully at the red part:

$$3\cos(2x) + 2\sin^2(x) = 0 \implies 3 \cdot {\color{red}2\cos^2(x) - 1} + 2\sin^2(x) = 0$$

$$\implies 6\cos^2(x) - 1 + 2\sin^2(x) = 0 \implies 2(3\cos^2 {\color{red}-1} + \sin^2(x)) = 0$$

$$\implies 3\cos^2(x) - 1 + 1 - \cos^2(x) = 0 \implies 2\cos^2(x) = 0 \implies \cos^2(x) = 0\cos(x) = 0$$

$$\implies x = \frac{\pi}{2} + k\pi \text{ (where } k \text{ is an integer)}$$

2.

$$3\cos(2x) + 2\sin^2(x) = 0 \implies 3(1 - \sin^2(x)) + 2\sin^2(x) = 0$$

$$\implies 3 - 3\sin^2(x) + 2\sin^2(x) = 0 \implies 3 - \sin^2(x) = 0 \implies \sin^2(x) = 3$$

$$\implies \sin(x) = \pm\sqrt{3} \implies \text{ no solution}$$

Need a hint? Look carefully at the red part:

$$3\cos(2x)+2\sin^2(x)=0 \implies 3(1-\sin^2(x))+2\sin^2(x)=0$$
$$\implies 3-3\sin^2(x)+2\sin^2(x)=0 \implies 3-\sin^2(x)=0 \implies \sin^2(x)=3$$
$$\implies \sin(x)=\pm\sqrt{3} \implies \text{no solution}$$

3.

$$3\cos(2x)+2\sin^2(x)=0 \implies 3(\cos^2(x)-1)+2(1-\cos^2(x))=0$$
$$\implies 6\cos^2(x)-3+2-2\cos^2(x)=0 \implies 4\cos^2(x)-1=0 \implies 4\cos^2(x)=-1$$
$$\implies \cos^2(x)=-\frac{1}{4} \implies \text{no real solutions}$$

Need a hint? Look carefully at the red part:

$$3\cos(2x)+2\sin^2(x)=0 \implies 3(\cos^2(x)-1)+2(1-\cos^2(x))=0$$
$$\implies 6\cos^2(x)-3+2-2\cos^2(x)=0 \implies 4\cos^2(x)-1=0 \implies 4\cos^2(x)=-1$$
$$\implies \cos^2(x)=-\frac{1}{4} \implies \text{no real solutions}$$

The Correction

$$3\cos(2x)+2\sin^2(x)=0 \implies 3(2\cos^2(x)-1)+2(1-\cos^2(x))=0$$
$$\implies 6\cos^2(x)-3+2-2\cos^2(x)=0 \implies 4\cos^2(x)=1 \implies \cos^2(x)=\frac{1}{4}$$
$$\implies \cos(x)=\pm\frac{1}{2} \implies x=\pm\frac{\pi}{3}+k\pi \text{ where } k \text{ is an integer}$$

(Roll the mouse over the area above to see the correction in blue)

Explanations

The first attempted solution has two mistakes. The first mistake occurs because parentheses were not used around the factor ($2\cos^2$(x)-1), which meant that the term 1 was not multiplied by the coefficient 3 in the next step. In the third step 2 was factored out of the entire left side incorrectly; factoring 2 out from 1 would leave 1/2, not 1.

The second mistake makes use of an incorrect formula for cos(2x); the coefficient 2 is missing from the $\sin^2$(x) term.

The third mistake has a simple algebra error; 1 was *added* to the left side of the equation but *subtracted* from the left side.

The keys to solving this problem are knowing the correct double angle formulas and then using correct algebra to solve the equation.