Common Trigonometry Mistakes

Example: Solve a trigonometric equation

Some problems provide the opportunity for more than one mistake.

The Goal

Solve the equation:

$$3\cos(2x) + 2\sin^2(x) = 0$$

The Mistakes

Find the mistakes:

1.

$$3\cos(2x) + 2\sin^2(x) = 0 \implies \cos(2x) + \sin^2(x) = 0 \implies 1 - 2\sin^2(x) + \sin^2(x) = 0$$

$$\implies 1 - \sin^2(x) = 0 \implies \sin^2(x) = 1 \implies \sin(x) = \pm 1 \implies x = \frac{\pi}{2} + k\pi$$

Need a hint? Look carefully at the red part:

$$3\cos(2x) + 2\sin^2(x) = 0 \implies \color{red}{\cos(2x) + \sin^2(x)} = 0 \implies 1 - 2\sin^2(x) + \sin^2(x) = 0$$

$$\implies 1 - \sin^2(x) = 0 \implies \sin^2(x) = 1 \implies \sin(x) = \pm 1 \implies x = \frac{\pi}{2} + k\pi$$

2.

$$3\cos(2x) + 2\sin^2(x) = 0 \implies 3\cos(2x) + 2\cos^2(x) - 1 = 0$$

$$\implies 3(2\cos^2(x) - 1) + 2\cos^2(x) - 1 = 0 \implies 6\cos^2(x) - 3 + 2\cos^2(x) - 1 = 0$$

$$\implies 8\cos^2(x) = 4 \implies \cos^2(x) = \frac{1}{2} \implies \cos(x) = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$$

$$\implies x = \frac{\pi}{4} + k\pi \text{ where } k \text{ is an integer}$$

Need a hint? Look carefully at the red part:

$$3\cos(2x)+2\sin^2(x)=0 \implies 3\cos(2x)+2\cos^2(x)-1=0$$

$$\implies 3(2\cos^2(x)-1)+2\cos^2(x)-1=0 \implies 6\cos^2(x)-3+2\cos^2(x)-1=0$$

$$\implies 8\cos^2(x)=4 \implies \cos^2(x)=\frac{1}{2} \implies \cos(x)=\pm\frac{1}{\sqrt{2}}=\pm\frac{\sqrt{2}}{2}$$

$$\implies x=\frac{\pi}{4}+k\pi \text{ where } k \text{ is an integer}$$

3.

$$3\cos(2x)+2\sin^2(x)=0 \implies 3(\cos^2(x)+\sin^2(x))+2\sin^2(x)=0$$

$$\implies 3\cos^2(x)+3\sin^2(x)+2\sin^2(x)=0 \implies 3\cos^2(x)+5\sin^2(x)=0$$

$$\implies 5\sin^2(x)=-3\cos^2(x) \implies \tan^2(x)=-\frac{3}{5} \implies \text{ no real solutions}$$

Need a hint? Look carefully at the red part:

$$3\cos(2x)+2\sin^2(x)=0 \implies 3(\cos^2(x)+\sin^2(x))+2\sin^2(x)=0$$

$$\implies 3\cos^2(x)+3\sin^2(x)+2\sin^2(x)=0 \implies 3\cos^2(x)+5\sin^2(x)=0$$

$$\implies 5\sin^2(x)=-3\cos^2(x) \implies \tan^2(x)=-\frac{3}{5} \implies \text{ no real solutions}$$

The Correction

$$3\cos(2x)+2\sin^2(x)=0 \implies 3(2\cos^2(x)-1)+2(1-\cos^2(x))=0$$

$$\implies 6\cos^2(x)-3+2-2\cos^2(x)=0 \implies 4\cos^2(x)=1 \implies \cos^2(x)=\frac{1}{4}$$

$$\implies \cos(x)=\pm\frac{1}{2} \implies x=\pm\frac{\pi}{3}+k\pi \text{ where } k \text{ is an integer}$$

Explanations

In the first mistake the coefficients of $\cos^2(2x)$ and $\sin^2(x)$ mysteriously vanished in the first step.

The second attempted solution makes use of an incorrect formula for cos(2x), in fact the negation of a correct formula.

In the third mistake once again an incorrect formula was used for cos(2x); recall that in fact $\cos^2(x) + \sin^2(x) = 1$, not cos(2x).

The keys to solving this problem are knowing the correct double angle formulas and then using correct algebra to solve the equation.