Common Trigonometry Mistakes

Example: Solve a trigonometric equation

Some problems provide the opportunity for more than one mistake.

The Goal

Solve the equation:

$$3\cos(2x)+2\sin^2(x)=0$$

The Mistakes

Find the mistakes:

1.

$$3\cos(2x)+2\sin^2(x)=0 \implies 3(2\sin^2(x)-1)(2x)+2\sin^2(x)=0$$
$$\implies 12\sin^2(x)-6x+2\sin^2(x)=0 \implies 14\sin^2(x)-6x=0 \implies ?$$

Need a hint? Look carefully at the red part:

$$3\cos(2x)+2\sin^2(x)=0 \implies {\color{red}3(2\sin^2(x)-1)(2x)}+2\sin^2(x)=0$$
$$\implies 12\sin^2(x)-6x+2\sin^2(x)=0 \implies 14\sin^2(x)-6x=0 \implies ?$$

2.

$$3\cos(2x)+2\sin^2(x)=0 \implies 3\left(\frac{1-\cos(2x)}{2}\right)+\frac{2(2\sin^2(x)\cos(x))}{1}=0$$

$$\implies 6(1-\cos(2x))+2(2\sin^2(x)\cos(x))=0 \implies 6\cos^2(x)+4(1-\cos^2(x))\cos(x)=0$$

$$\implies 6\cos^2(x)+4\cos(x)-4\cos^3(x)=0 \implies \cos(x)(6\cos(x)+4-4\cos^2(x))=0$$

$$\implies -2\cos(x)(2\cos^2(x)-3\cos(x)-2)=0 \implies \cos(x)(2\cos(x)+1)(\cos(x)-2)=0$$

$$\implies \cos(x)=0 \text{ or } \cos(x)=-\frac{1}{2} \text{ or } \cos(x)=2$$

$$\implies x=\frac{\pi}{2}+2k\pi \text{ or } x=\pm\frac{\pi}{3}+2k\pi \text{ where } k \text{ is an integer}$$

Need a hint? Look carefully at the red part:

$$3\cos(2x)+2\sin^2(x)=0 \implies 3\left(\frac{1-\cos(2x)}{2}\right)+\frac{2(2\sin^2(x)\cos(x))}{1}=0$$

$$\implies 6(1-\cos(2x))+2(2\sin^2(x)\cos(x))=0 \implies 6\cos^2(x)+4(1-\cos^2(x))\cos(x)=0$$

$$\implies 6\cos^2(x)+4\cos(x)-4\cos^3(x)=0 \implies \cos(x)(6\cos(x)+4-4\cos^2(x))=0$$

$$\implies -2\cos(x)(2\cos^2(x)-3\cos(x)-2)=0 \implies \cos(x)(2\cos(x)+1)(\cos(x)-2)=0$$

$$\implies \cos(x)=0 \text{ or } \cos(x)=-\frac{1}{2} \text{ or } \cos(x)=2$$

$$\implies x=\frac{\pi}{2}+2k\pi \text{ or } x=\pm\frac{\pi}{3}+2k\pi \text{ where } k \text{ is an integer}$$

3.

$$3\cos(2x)+2\sin^2(x)=0 \implies 3\cos(2x)+2(1-\cos^2(x))=0$$
$$\implies 3(1+\cos^2(x))+2(1-\cos^2(x))=0 \implies 3+3\cos^2(x)+2-2\cos^2(x)=0$$
$$\implies 5+\cos^2(x)=0 \implies \cos^2(x)=5 \implies \cos(x)=\pm\sqrt{5} \implies \text{no solution}$$

Need a hint? Look carefully at the red part:

$$3\cos(2x)+2\sin^2(x)=0 \implies 3\cos(2x)+2(1-\cos^2(x))=0$$
$$\implies 3(1+\cos^2(x))+2(1-\cos^2(x))=0 \implies 3+3\cos^2(x)+2-2\cos^2(x)=0$$
$$\implies 5+\cos^2(x)=0 \implies \cos^2(x)=5 \implies \cos(x)=\pm\sqrt{5} \implies \text{no solution}$$

The Correction

$$3\cos(2x)+2\sin^2(x)=0 \implies 3(2\cos^2(x)-1)+2(1-\cos^2(x))=0$$

$$\implies 6\cos^2(x)-3+2-2\cos^2(x)=0 \implies 4\cos^2(x)=1 \implies \cos^2(x)=\frac{1}{4}$$

$$\implies \cos(x)=\pm\frac{1}{2} \implies x=\pm\frac{\pi}{3}+k\pi \text{ where } k \text{ is an integer}$$

Explanations

In the first mistake an incorrect formula was used for cos(2x). Eventually the equation reduces to a form that we cannot solve algebraically.

The second attempted solution has three mistakes, the first and third of which involve the use of incorrect formulas for cos(2x). The second mistake is the replacement of the fraction

3/2 by 6 (for some reason the *product* of 3 and 2 was computed).

In the third mistake once again an incorrect formula was used for cos(2x).

The keys to solving this problem are knowing the correct double angle formulas and then using correct algebra to solve the equation.