Common Trigonometry Mistakes

Example: Simplifying a trigonometric expression

Some problems provide the opportunity for more than one mistake.

The Goal

Simplify the expression:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)}$$

The Mistakes

Find the mistakes:

1.

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{2\cos^2(x)-1}{\cos(x)} = \frac{2\sin(x)\cos^2(x)}{\sin(x)\cos(x)} - \frac{2\sin(x)\cos^2(x)-1}{\sin(x)\cos(x)}$$

$$= \frac{2\sin(x)\cos^2(x) - 2\sin(x)\cos^2(x)-1}{\sin(x)\cos(x)} = \frac{-1}{\sin(x)\cos(x)} = \sec(x)\csc(x)$$

Need a hint? Look carefully at the red parts:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{2\cos^2(x)-1}{\cos(x)} = \frac{2\sin(x)\cos^2(x)}{\sin(x)\cos(x)} - \frac{2\sin(x)\cos^2(x){\color{red}-1}}{\sin(x)\cos(x)}$$

$$= \frac{2\sin(x)\cos^2(x) - 2\sin(x)\cos^2(x){\color{red}-1}}{\sin(x)\cos(x)} = \frac{-1}{\sin(x)\cos(x)} = \sec(x)\csc(x)$$

2.

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{2\cos^2(x)-1}{\cos(x)}$$

$$= \frac{2\sin(x)\cos^2(x)}{\sin(x)\cos(x)} - \frac{(2\cos^2(x)-1)\sin(x)}{\sin(x)\cos(x)} = \frac{2\cos^2(x)\sin(x) - (2\cos^2(x)-1)\cancel{\sin(x)}}{\cancel{\sin(x)}\cos(x)}$$

$$= \frac{\cancel{2\cos^2(x)}\sin(x) - (\cancel{2\cos^2(x)}+1)}{\cos(x)} = \frac{1}{\cos(x)} = \sec(x)$$

Need a hint? Look carefully at the red and green parts:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{2\cos^2(x)-1}{\cos(x)}$$

$$= \frac{2\sin(x)\cos^2(x)}{\sin(x)\cos(x)} - \frac{(2\cos^2(x)-1)\sin(x)}{\sin(x)\cos(x)} = \frac{2\cos^2(x)\sin(x) - (2\cos^2(x)-1)\cancel{\sin(x)}}{\cancel{\sin(x)}\cos(x)}$$

$$= \frac{2\cancel{\cos^2(x)}\sin(x) - (2\cancel{\cos^2(x)}+1)}{\cos(x)} = \frac{1}{\cos(x)} = \sec(x)$$

The Correction

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{2\cos^2(x)-1}{\cos(x)}$$

$$= \frac{2\sin(x)\cos^2(x)}{\sin(x)\cos(x)} - \frac{(2\cos^2(x)-1)\sin(x)}{\sin(x)\cos(x)} = \frac{2\cos^2(x)\sin(x) - 2\cos^2(x)\sin(x) + \sin(x)}{\sin(x)\cos(x)}$$

$$= \frac{\sin(x)}{\sin(x)\cos(x)} = \frac{1}{\cos(x)} = \sec(x)$$

Explanations

The first attempted solution has two mistakes; in the second step the second fraction's numerator should have had *all* terms multiplied by sin(x). In the third step the minus sign was not distributed correctly.

Two incorrect cancellations mar the second attempted solution. A multiplicatively cancelled part must be a *common factor* of numerator and denominator (first mistake). An additively cancelled part must be *exactly the same term* (second mistake). Note sin(x) *is* a factor of all terms in the numerator, and should have been cancelled from all terms (first error). For the second error, $2\cos^2(x)\sin(x)$ is *not* equal to $2\cos^2(x)$. There's also a sign change mistake shown in green.

The keys to solving this problem are knowing the correct double angle formulas (correct in both of these attempted solutions) and then using careful algebra to complete the simplification.