Common Trigonometry Mistakes

Example: Simplifying a trigonometric expression

Some problems provide the opportunity for more than one mistake.

The Goal

Simplify the expression:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)}$$

The Mistakes

Find the mistakes:

1.

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{\cos^2(x) - \sin^2(x)}{\cos(x)}$$

$$= 2\cos(x) - \cos(x) - \sin^2(x) = \cos(x) - \sin^2(x)$$

Need a hint? Look carefully at the red part:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{\cos^2(x) - \sin^2(x)}{\cos(x)}$$

$$= 2\cos(x) - \cos(x) - \sin^2(x) = \cos(x) - \sin^2(x)$$

2.

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{1 - 2\sin^2(x)}{\cos(x)}$$

$$= 2\cos(x) + \frac{-1 + 2(1 - \cos^2(x))}{\cos(x)} = 2\cos(x) - 1 + 2 - 2\cos(x) = 1$$

Need a hint? Look carefully at the red part:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{1 - 2\sin^2(x)}{\cos(x)}$$

$$= 2\cos(x) + \frac{-1 + 2(1 - \cos^2(x))}{\cos(x)} = 2\cos(x) - 1 + 2 - 2\cos(x) = 1$$

3.

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{2\cos^2(x)-1}{\cos(x)}$$

$$= \frac{2\sin(x)\cos^2(x) - 2\sin(x)\cos^2(x) - \sin(x)}{\sin(x)\cos(x)} = \frac{-\sin(x)}{\sin(x)\cos(x)} = \frac{-1}{\cos(x)} = -\sec(x)$$

Need a hint? Look carefully at the red part:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{2\cos^2(x)-1}{\cos(x)}$$

$$= \frac{2\sin(x)\cos^2(x) - 2\sin(x)\cos^2(x) - \sin(x)}{\sin(x)\cos(x)} = \frac{-\sin(x)}{\sin(x)\cos(x)} = \frac{-1}{\cos(x)} = -\sec(x)$$

The Correction

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{2\cos^2(x)-1}{\cos(x)}$$

$$= 2\cos(x) - 2\cos(x) + \frac{1}{\cos(x)} = \frac{1}{\cos(x)} = \sec(x)$$

Explanations

In the first mistake the fraction is reduced incorrectly; there still should be a $\sin^2(x)/\cos(x)$ term. Perhaps the error was the result of attempting to cancel cos(x) from numerator and denominator; however, cos(x) is not a *common factor* of the numerator, so cannot be cancelled. In addition, the minus sign was not correctly distributed across the fraction

In the second attempted solution incorrect cancellation is definitely the culprit. The cos(x) is *not* a common factor of the numerator, so cannot be cancelled.

The third mistake has a simple sign error - the minus sign was not correctly distributed across the fraction. The final term in the numerator should be + sin(x).

The keys to solving this problem are knowing the correct double angle formulas (correct in all three of these attempted solutions) and then using careful algebra to complete the simplification.