Common Trigonometry Mistakes
Example: Simplifying a trigonometric expression

Some problems provide the opportunity for more than one mistake.

The Goal

Simplify the expression:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)}$$

The Mistakes

Find the mistakes:

1.

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} \cdot \sin(x) - \frac{2\cos^2(x) - 1}{\cos(x)} \cdot \cos(x)$$

$$= \frac{1}{\cos(x)}(\sin(x)\cos(x)) = \sin(x)$$

Need a hint? Look carefully at the red part:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} \cdot \color{red}{\sin(x)} - \frac{2\cos^2(x) - 1}{\cos(x)} \cdot \color{red}{\cos(x)}$$

$$= \frac{1}{\cos(x)}(\sin(x)\cos(x)) = \sin(x)$$

2.

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{\cos^2(x) - 1}{\cos(x)}$$

$$= 2\cos(x) - \cos(x) + \frac{1}{\cos(x)} = \cos(x) + \sec(x)$$

Need a hint? Look carefully at the red part:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{\color{red}{?\cdot}\cos^2(x) - 1}{\cos(x)}$$

$$= 2\cos(x) - \cos(x) + \frac{1}{\cos(x)} = \cos(x) + \sec(x)$$

3.

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{1+2\cos^2(x)}{\cos(x)}$$

$$= 2\cos(x) - \frac{-1+2\cos^2(x)}{\cos(x)} = \frac{2\cos^2(x)-1-2\cos^2(x)}{\cos(x)} = \frac{1}{\cos(x)} = \sec(x)$$

Need a hint? Look carefully at the red part:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{1+2\cos^2(x)}{\cos(x)}$$

$$= 2\cos(x) - \frac{-1+2\cos^2(x)}{\cos(x)} = \frac{2\cos^2(x)-1-2\cos^2(x)}{\cos(x)} = \frac{1}{\cos(x)} = \sec(x)$$

The Correction

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{2\cos^2(x)-1}{\cos(x)} = 2\cos(x) - \frac{2\cos^2(x)-1}{\cos(x)}$$

$$= \frac{2\cos^2(x)-2\cos^2(x)+1}{\cos(x)} = \frac{1}{\cos(x)} = \sec(x)$$

Explanations

In the first mistake the student attempts to clear the denominators by introducing the sin(x) and cos(x) factors, completely incorrectly. The second step is a complete mystery.

In the second attempted solution coefficient 2 is missing from the cosine double-angle formula.

In the third mistake cosine double-angle formula is incorrect.

The keys to solving this problem are knowing the correct double angle formulas and then using careful algebra to complete the simplification.