Common Trigonometry Mistakes

Example: Simplifying a trigonometric expression

Some problems provide the opportunity for more than one mistake.

The Goal

Simplify the expression:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)}$$

The Mistakes

Find the mistakes:

1.

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(y)}{\sin(x)} - \frac{2\cos^2(x)-1}{\cos(x)} = \sin(x^2) - 1 = \sec(x)$$

Need a hint? Look carefully at the red part:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(y)}{\sin(x)} - \frac{2\cos^2(x)-1}{\cos(x)} = \sin(x^2) - 1 = \sec(x)$$

2.

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{1-2\sin^2(x)}{\sin(x)} - \frac{2\cos^2(x)-1}{\cos(x)}$$

$$= \frac{\cos(x) - 2\sin^2(x)\cos(x) - 2\cos^2(x)\sin(x) + \sin(x)}{\sin(x)\cos(x)}$$

$$= \frac{(\sin(x)+\cos(x)) - 2\sin(x)\cos(x)(\sin(x)+\cos(x))}{\sin(x)\cos(x)}$$

$$= \frac{(\sin(x)+\cos(x))(1-2\sin(x)\cos(x))}{\sin(x)\cos(x)} = ?$$

Need a hint? Look carefully at the red part:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{1-2\sin^2(x)}{\sin(x)} - \frac{2\cos^2(x)-1}{\cos(x)}$$

$$= \frac{\cos(x) - 2\sin^2(x)\cos(x) - 2\cos^2(x)\sin(x) + \sin(x)}{\sin(x)\cos(x)}$$

$$= \frac{(\sin(x)+\cos(x)) - 2\sin(x)\cos(x)(\sin(x)+\cos(x))}{\sin(x)\cos(x)}$$

$$= \frac{(\sin(x)+\cos(x))(1-2\sin(x)\cos(x))}{\sin(x)\cos(x)} = ?$$

3.

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{\sin(x)\cos(x)}{\sin(x)} - \frac{2\cos^2(x)-1}{\cos(x)} = \cos(x) - 2\cos(x) - 1 = -\cos(x) - 1$$

Need a hint? Look carefully at the red part:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{\sin(x)\cos(x)}{\sin(x)} - \frac{2\cos^2(x)-1}{\cos(x)} = \cos(x) - 2\cos(x) - 1 = -\cos(x) - 1$$

The Correction

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{2\cos^2(x)-1}{\cos(x)}$$

$$= 2\cos(x) - 2\cos(x) + \frac{1}{\cos(x)} = \frac{1}{\cos(x)} = \sec(x)$$

Explanations

The first mistake contains mistakes in all three steps, each without obvious explanation. In the first step the variable y makes a cameo appearance. I see no logic to the second step, nor the third step, other than wishful thinking.

In the second attempted solution the wrong formula were used for sin(2x); the formula used is one of the variations for cos(2x).

In the third mistake the formula for sin(2x) is missing the coefficient "2". In the second step the quotient has been reduced incorrectly - the second term should be positive 1/cos(x), not -1. Take care when distributing a minus sign over an expression!

The keys to solving this problem are knowing the correct double angle formulas and then using careful algebra to complete the simplification.

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