
Common Trigonometry Mistakes

Example: Simplifying a trigonometric expression

Some problems provide the opportunity for more than one mistake.

The Goal

Simplify the expression:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)}$$

The Mistakes

Find the mistakes:

1.

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \sin(x) - \cos(x) = \sec(x)$$

Need a hint? Look carefully at the red part:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \sin(x) - \cos(x) = \sec(x)$$

2.

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{1+\cos(2x)}{2} - \frac{1-\cos(2x)}{2} = \sin(x) - \cos(x) = ?$$

Need a hint? Look carefully at the red part:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{1+\cos(2x)}{2} - \frac{1-\cos(2x)}{2} = \sin(x) - \cos(x) = ?$$

3.

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{2\cos^2(x-1)}{\cos(x)} = 2\cos(x) - 2\cos^2(x-1) = ?$$

Need a hint? Look carefully at the red part:

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{2\cos^2(x-1)}{\cos(x)} = 2\cos(x) - 2\cos^2(x-1) = ?$$

The Correction

$$\begin{aligned}\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} &= \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{2\cos^2(x) - 1}{\cos(x)} \\ &= 2\cos(x) - 2\cos(x) + \frac{1}{\cos(x)} = \frac{1}{\cos(x)} = \sec(x)\end{aligned}$$

Explanations

In the first mistake it appears that $\sin(2x)$ was interpreted as $\sin(x)\sin(x)$, which is false; likewise for $\cos(2x)$. $\sin(x)$ and $\cos(x)$ were then cancelled. The second step makes no sense, but perhaps was motivated by the expected answer ($\sec(x)$).

In the second attempted solution the wrong formulas were used for $\sin(2x)$ and $\cos(2x)$; there seems to be confusion with the formulas for $\sin^2(x)$ and $\cos^2(x)$; even these are not correct. The second step is pure wishful thinking, but even that does not help.

In the third mistake the formula for $\cos(2x)$ is misrecalled, with the parentheses misplaced. The mistake is compounded with an incorrect cancellation in the second step - a denominator is simply erased.

The keys to solving this problem are knowing the correct double angle formulas and then using correct algebra to complete the simplification.

[<-- Back](#)