Common Trigonometry Mistakes
Example: Simplifying an Expression

The Goal

Simplify the following:

$$\frac{\cos(2x)}{\cos(x)}$$

The Mistake

Find the mistake:

$$\frac{\cos(2x)}{\cos(x)} = \frac{\cos}{1} = \cos$$

Need a hint? Look carefully at the red part:

$$\frac{\cos(2x)}{\cos(x)} = \frac{\cos}{1} = \cos$$

The Correction

$$\frac{\cos(2x)}{\cos(x)} = \frac{\cos^2(x) - \sin^2(x)}{\cos(x)} = \frac{\cos^2(x)}{\cos(x)} - \sin(x)\frac{\sin(x)}{\cos(x)} = \cos(x) - \sin(x)\tan(x)$$

An Explanation

This is an example of awful cancellation that results from not understanding the function notation of trigonometric functions. Cosine always comes with an *argument*, that is, an *input*. Cosine is a function! Think about the square root function. We say √x as "square root *of* x", not "square root *times* x", for a good reason. We plug x *into* the square root function. It is meaningless to talk about multiplying the square root symbol by anything. In the same way, when we write cos(2x) we mean 2x *plugged into* the cosine function. Cosine by itself has no meaning without an argument! And since the "cos" does not represent a *factor* in the expression of this example, it *cannot* be cancelled.

It's not exactly clear how the mistake was made. Perhaps the first mistake was misinterpreting cos(2x) as $\cos^2(x)$, which then led to the bogus cancellation of a "cos" and an x from both numerator and denominator. Neither cancellation is legal.

<-- Back