Common Trigonometry Mistakes
Example: Law of Sines

The Goal

Approximate (to two decimal place accuracy) the remaining parts of all possible triangles ABC with α = 54°, a = 6 and b=7 (where the side a is opposite the angle α, the side b is opposite the angle β and the side c is opposite the angle γ).

The Mistake

Find the mistake:

By the Law of Sines,

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} \implies \frac{\sin(54^\circ)}{6} = \frac{\sin(\beta)}{7} \implies \sin(\beta) = \frac{7\sin(54^\circ)}{6} \approx 0.94385$$

$$\implies \beta \approx \sin^{-1}(0.94385) \approx 70.71^\circ$$

Since the angles in a triangle add to 180°, the third angle γ ≈ 180° - 54° - 70.71° = 55.29°. Apply the Law of Sines a second time to find c:

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\gamma)}{c} \implies \frac{\sin(54^\circ)}{6} = \frac{\sin(55.29^\circ)}{c} \implies c = \frac{6\sin(55.29^\circ)}{\sin(54^\circ)} \approx 6.10$$

We conclude that the remaining parts of the triangle are β ≈ 70.71°, γ ≈ 55.29° and c ≈ 6.10.

Need a hint to find the mistake? Look carefully at the red part from the first calculation:

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} \implies \frac{\sin(54^\circ)}{6} = \frac{\sin(\beta)}{7} \implies \sin(\beta) = \frac{7\sin(54^\circ)}{6} \approx 0.94385$$

$$\implies \beta \approx \sin^{-1}(0.94385) \approx 70.71^\circ \text{ or ?}$$

The Correction

By the Law of Sines,

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} \implies \frac{\sin(54^\circ)}{6} = \frac{\sin(\beta)}{7} \implies \sin(\beta) = \frac{7\sin(54^\circ)}{6} \approx 0.94385$$

$$\implies \beta \approx \sin^{-1}(0.94385) \approx 70.71^\circ \text{ or } 180^\circ - 70.71^\circ = 109.29^\circ$$

Therefore it is possible that there are *two triangles*, one of which may have an *obtuse* angle β. Consider each possibilty.

I. If β ≈ 70.71°, then (as above) the third angle γ ≈ 180° - 54° - 70.71° = 55.29°. Apply the Law of Sines to find c:

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\gamma)}{c} \implies \frac{\sin(54^\circ)}{6} = \frac{\sin(55.29^\circ)}{c} \implies c = \frac{6\sin(55.29^\circ)}{\sin(54^\circ)} \approx 6.10$$

The remaining parts of the triangle in this case are β ≈ 70.71°, γ ≈ 55.29° and c ≈ 6.10.

II. If β ≈ 109.29°, then the third angle γ ≈ 180° - 54° - 109.29° = 16.71°. Since the angle γ turns out to be positive, there *is* a second triangle. Once again, apply the Law of Sines to find c in this case:

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\gamma)}{c} \Longrightarrow \frac{\sin(54^\circ)}{6} = \frac{\sin(6.71)}{c} \Longrightarrow c = \frac{6\sin(16.71^\circ)}{\sin(54^\circ)} \approx 2.13$$

The remaining parts of the triangle in this case are β ≈ 109.29°, γ ≈ 16.71° and c ≈ 2.13.

An Explanation

The key in problems using the Law of Sines is to remember that there are *two* angles between 0° and 180° whose sines have the same value. If θ is an angle in the first quadrant, then the angle in the second quadrant with the same value for the sine function is 180° - θ.