Common Trigonometry Mistakes
Example: Solving an Equation

The Goal

Solve the following equation for t:

$$\sin(\frac{\pi}{6}t) = 0 \text{ with } 0 \le t \le 24$$

The Mistake

Find the mistake:

$$\sin(\frac{\pi}{6}t) = 0 \implies \sin(\frac{\pi}{6})\sin(t) = 0 \implies \frac{1}{2}\sin(t) = 0 \implies \sin(t) = 0$$
$$\implies t = k\pi \text{ for } k \text{ an integer} \implies t = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, 7\pi$$

Need a hint? Look carefully at the red part:

$$\sin(\frac{\pi}{6}t) = 0 \implies \sin(\frac{\pi}{6})\sin(t) = 0 \implies \frac{1}{2}\sin(t) = 0 \implies \sin(t) = 0$$
$$\implies t = k\pi \text{ for } k \text{ an integer} \implies t = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, 7\pi$$

The Correction

$$\sin(\frac{\pi}{6}t) = 0 \implies \frac{\pi}{6}t = k\pi \text{ for k an integer} \implies t = 6k \text{ for k an integer}$$
$$\implies t = 0, 6, 12, 18, 24$$

An Explanation

It is *not true* that sin(AB) is equal to sin(A)sin(B) - there is *no* nice formula for sin(AB). The correct method is to set the argument of the sine, namely (π/6)t, equal to the angles for which sine takes the value 0, namely all integer multiples of π. It is then easy to solve the resulting equation and identify the solutions for which 0 ≤ t ≤ 24.
(There *is* a formula for sin(A)sin(B):

$$\sin(A)\sin(B) = \frac{1}{2}(\cos(A-B) - \cos(A+B))$$

This formula is one of the *prosthaphaeretic* formulas.)

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