Common Calculus Mistakes
Example: Chain Rule

The Goal

Find

$$\frac{d}{dt}e^{t^2}$$

The Mistake

Find the mistake:

$$\frac{d}{dt}e^{t^2} = e^t e^{t^2}$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dt}e^{t^2} = \color{red}{e^t} e^{t^2}$$

The Correction

$$\frac{d}{dt}e^{t^2} = \color{blue}{2t} e^{t^2}$$

An Explanation

When the *chain rule* is applied to the composition of the (natural) exponential function f(t) = e^t and the function g(t), the result is:

$$\frac{d}{dt}f(g(t)) = f'(g(t))g'(t) \implies \frac{d}{dt}e^{g(t)} = e^{g(t)}g'(t)$$

In this case g(t) = t^2, so g'(t) = 2t.
It is usual to write the 2t factor in front of the exponential in the answer.