Common Calculus Mistakes
Derivatives of Hyperbolic Functions: Hyperbolic Cosine

Some problems provide the opportunity for more than one mistake.

The Goal

Find

$$\frac{d}{dx}\cosh(x)$$

The Mistakes

Find the mistakes:

1.

$$\frac{d}{dx}\cosh(x) = -\sinh(x)$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dx}\cosh(x) = -\sinh(x)$$

2.

$$\frac{d}{dx}\cosh(x) = \frac{e^x + e^{-x}}{2}$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dx}\cosh(x) = \frac{e^x + e^{-x}}{2}$$

A Correct Solution

$$\frac{d}{dx}\cosh(x) = \sinh(x)$$

Explanations

In both cases the formula for the derivative is wrong. The first mistake mimics the pattern for

the trigonometric functions - but the derivatives of the hyperbolic functions don't agree with the corresponding trigonometric derivatives in sign in all cases. In the second mistake the student has confused the *definition* of hyperbolic cosine (in terms of exponentials) with the *derivative* of hyperbolic cosine.

Basic derivative formulas should be learned. For hyperbolic cosine there are two choices: memorize the correct derivative (try the flashcards at Calculus Facts); or derive the derivative using the definition of hyperbolic cosine whenever this derivative is needed:

$$\frac{d}{dx}\cosh(x) = \frac{d}{dx}\frac{e^x + e^{-x}}{2} = \frac{1}{2}\left(\frac{d}{dx}e^x + \frac{d}{dx}e^{-x}\right) = \frac{1}{2}(e^x - e^{-x}) = \sinh(x)$$