Common Calculus Mistakes

Quotient Rule

The Goal

Find

$$\frac{d}{dx}\frac{\tan(x)}{\sqrt{x}}$$

The Mistake

Find the mistake:

$$\frac{d}{dx}\frac{\tan(x)}{\sqrt{x}} = \frac{x^{\frac{1}{2}}\left(\frac{1}{\cos^2(x)}\right) - \tan(x)\frac{1}{2}x^{-\frac{1}{2}}}{(\sqrt{x})^2} = \frac{\sqrt{x} - \tan(x)}{2x\sqrt{x}\cos^2(x)}$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dx}\frac{\tan(x)}{\sqrt{x}} = \frac{x^{\frac{1}{2}}\left(\frac{1}{\cos^2(x)}\right) - \tan(x)\frac{1}{2}x^{-\frac{1}{2}}}{(\sqrt{x})^2} = \frac{\sqrt{x} - \tan(x)}{2x\sqrt{x}\cos^2(x)}$$

The Correction

$$\frac{d}{dx}\frac{\tan(x)}{\sqrt{x}} = \frac{x^{\frac{1}{2}}\left(\frac{1}{\cos^2(x)}\right) - \tan(x)\frac{1}{2}x^{-\frac{1}{2}}}{(\sqrt{x})^2} = \frac{\frac{1}{2}x^{-\frac{1}{2}}\frac{1}{\cos^2(x)}(2x - \sin(x)\cos(x))}{x}$$

$$= \frac{2x - \sin(x)\cos(x)}{2x\sqrt{x}\cos^2(x)}$$

An Explanation

The quotient rule is used correctly, but the algebra used to simplify the result is incorrect. Only *common* factors of the two terms in the numerator can be written in the denominator the way the student wanted. "The Correction" shows how to correctly factor out these quantities.