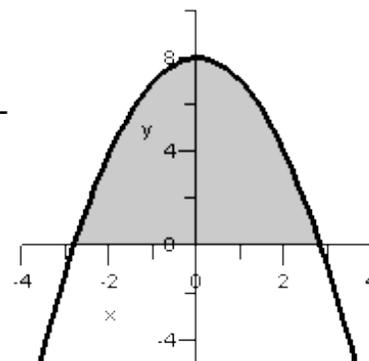

Common Calculus Mistakes

Area under a curve

The Goal

Find the exact area under the curve $y = 8 - x^2$ and above the x -axis.



The Mistake

Find the mistake:

$y = 8 - x^2$ intersects the x -axis at $x = \pm\sqrt{8} = \pm 2.82$, so area

$$A = \int_{-2.82}^{2.82} 8 - x^2 dx = \left(8x - \frac{1}{3}x^3\right) \Big|_{-2.82}^{2.82}$$
$$= 15.08 - (-15.08) = 30.16$$

Need a hint? Look carefully at the red part:

$y = 8 - x^2$ intersects the x -axis at $x = \pm\sqrt{8} = \pm 2.82$, so area

$$A = \int_{-2.82}^{2.82} 8 - x^2 dx = \left(8x - \frac{1}{3}x^3\right) \Big|_{-2.82}^{2.82}$$
$$= 15.08 - (-15.08) = 30.16$$

The Correction

$y = 8 - x^2$ intersects the x -axis at $x = \pm\sqrt{8} (\approx \pm 2.82)$, so area

$$A = \int_{-\sqrt{8}}^{\sqrt{8}} 8 - x^2 dx = \left(8x - \frac{1}{3}x^3\right) \Big|_{-\sqrt{8}}^{\sqrt{8}} = \left(8\sqrt{8} - \frac{1}{3}8\sqrt{8}\right) - \left(-8\sqrt{8} + \frac{1}{3}8\sqrt{8}\right)$$
$$= \frac{16}{3}\sqrt{8} - \left(-\frac{16}{3}\sqrt{8}\right) = \frac{32\sqrt{8}}{3} = \frac{64\sqrt{2}}{3}$$

An Explanation

The mistake was to *approximate* $\sqrt{8}$ by 2.82. The instructions were to find the *exact* value of the area, so the problem should be worked out with the exact values. This problem is one for which your calculator should be left turned *off*.