Common Calculus Mistakes

Derivative of inverse sine

The Goal

Find

$$\frac{d}{dx}(\sin^{-1}(x))$$

The Mistake

Find the mistake:

$$\frac{d}{dx}(\sin^{-1}(x)) = -\sin^{-2}(x)\cos(x)$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dx}(\sin^{-1}(x)) = -\sin^{-2}(x)\cos(x)$$

The Correction

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

An Explanation

The notation $\sin^{-1}(x)$ has been misunderstood to mean 1/sin(x). That interpretation might seem to be a natural extension of notation such as $\sin^2(x)$, which does mean $(\sin(x))^2$, and $\sin^{-2}(x)$, which does mean $1/(\sin(x))^2$. However, for any function f(x), *by convention* when we write $f^{-1}(x)$, we mean instead the *inverse function* for f(x). So $\sin^{-1}(x)$ means *the inverse sine of x*, that is, the function that *undoes* the sine function. It is *not* equal to 1/sin(x). The inverse sine function has the derivative shown in "The Correction".