Common Calculus Mistakes
Derivative of an Integral

Some problems provide the opportunity for more than one mistake.

The Goal

Find the derivative F'(x), where

$$F(x) = \int_2^x \sin\left(\frac{1}{\sqrt{t^2+1}}\right) dt$$

The Mistakes

Find the mistakes:

1.

$$F'(x) = \int_2^x -\cos\left(\frac{1}{\sqrt{t^2+1}}\right) dt$$

Need a hint? Look carefully at the red part:

$$F'(x) = \int_2^x -\cos\left(\frac{1}{\sqrt{t^2+1}}\right) dt$$

2.

$$F'(x) = \cos\left(\frac{1}{\sqrt{t^2+1}}\right) \frac{1}{2\sqrt{t^2+1}}(2t) = \cos\left(\frac{1}{\sqrt{t^2+1}}\right) \frac{t}{\sqrt{t^2+1}}$$

Need a hint? Look carefully at the red part:

$$F'(x) = \cos\left(\frac{1}{\sqrt{t^2+1}}\right) \frac{1}{2\sqrt{t^2+1}}(2t) = \cos\left(\frac{1}{\sqrt{t^2+1}}\right) \frac{t}{\sqrt{t^2+1}}$$

3.

$$F'(x) = \sin\left(\frac{1}{\sqrt{t^2+1}}\right)$$

Need a hint? Look carefully at the red part:

$$F'(x) = \sin\left(\frac{1}{\sqrt{t^2+1}}\right)$$

4.

$$F'(x) = \frac{1}{\sqrt{t^2+1}}$$

Need a hint? Look carefully at the red part:

$$F'(x) = \frac{1}{\sqrt{t^2+1}}$$

5.

$$F'(x) = \frac{1}{\sqrt{(t^2+1)+1}}\frac{1}{2}(t^2+1)^{-\frac{1}{2}}2t\,\Big|_2^{|x} = \frac{x}{\sqrt{(x^2+1)+1}\sqrt{x^2+1}} - \frac{2}{\sqrt{6}\sqrt{5}}$$

Need a hint? Look carefully at the red part:

$$F'(x) = \frac{1}{\sqrt{(t^2+1)+1}}\frac{1}{2}(t^2+1)^{-\frac{1}{2}}2t\,\Big|_2^{|x} = \frac{x}{\sqrt{(x^2+1)+1}\sqrt{x^2+1}} - \frac{2}{\sqrt{6}\sqrt{5}}$$

6.

$$F'(x) = -\cos\left(\frac{1}{\sqrt{t^2+1}}\right)\left(-\frac{1}{2}\right)(t^2+1)^{-\frac{3}{2}}(2t)(2)\,\Big|_2^{|x}$$

$$= \frac{2x}{(x^2+1)^{\frac{3}{2}}}\cos\left(\frac{1}{\sqrt{x^2+1}}\right) - \frac{4}{(5)^{\frac{3}{2}}}\cos\left(\frac{1}{\sqrt{5}}\right)$$

Need a hint? Look carefully at the red part:

$$F'(x) = -\cos\left(\frac{1}{\sqrt{t^2+1}}\right)\left(-\frac{1}{2}\right)(t^2+1)^{-\frac{3}{2}}(2t)(2)\,\Big|_2^{|x}$$

$$= \frac{2x}{(x^2+1)^{\frac{3}{2}}}\cos\left(\frac{1}{\sqrt{x^2+1}}\right) - \frac{4}{(5)^{\frac{3}{2}}}\cos\left(\frac{1}{\sqrt{5}}\right)$$

7.

$$F'(x) = -\cos\left(2\left(\frac{1}{3}t^3+t\right)^{\frac{1}{2}}\right) + C$$

Need a hint? Look carefully at the red part:

$$F'(x) = -\cos\left(2\left(\frac{1}{3}t^3+t\right)^{\frac{1}{2}}\right) + C$$

A Correct Solution

$$F'(x) = \sin\left(\frac{1}{\sqrt{x^2+1}}\right)$$

Explanations

By what is often called the Second Fundamental Theorem of Calculus,

$$F(x) = \int_2^x f(t)\,dt \implies F'(x) = f(x)$$

so all that needs to be done is to write down the integrand (the function inside the integral) with the variable t replaced by x. All of the attempted solutions are pure fiction, although the only error in Mistake 4 is that it uses the incorrect variable.