Common Calculus Mistakes
Compound Chain Rule

Some problems provide the opportunity for more than one mistake.

The Goal

Find

$$\frac{d}{dt}\sqrt{\sin(e^t)}$$

The Mistakes

Find the mistakes:

1.

$$\frac{d}{dt}\sqrt{\sin(e^t)} = \frac{1}{2\sqrt{\sin(e^t)}}\ e^t$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dt}\sqrt{\sin(e^t)} = \frac{1}{2\sqrt{\sin(e^t)}}?\cdot e^t$$

2.

$$\frac{d}{dt}\sqrt{\sin(e^t)} = \frac{1}{2}(\cos(e^t))^{-\frac{1}{2}}e^t$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dt}\sqrt{\sin(e^t)} = \frac{1}{2}(\cos(e^t))^{-\frac{1}{2}}e^t$$

3.

$$\frac{d}{dt}\sqrt{\sin(e^t)} = \frac{1}{2}(\sin(e^t))^{-\frac{1}{2}}\cos(e^t)$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dt}\sqrt{\sin(e^t)} = \frac{1}{2}(\sin(e^t))^{-\frac{1}{2}}\cos(e^t)?$$

4.

$$\frac{d}{dt}\sqrt{\sin(e^t)} = \left(\frac{1}{2}\cos(e^t)\right)^{-\frac{1}{2}}e^t$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dt}\sqrt{\sin(e^t)} = \left(\frac{1}{2}\cos(e^t)\right)^{-\frac{1}{2}}e^t$$

A Correct Solution

$$\frac{d}{dt}\sqrt{\sin(e^t)} = \frac{1}{2}(\sin(e^t))^{-\frac{1}{2}}\cos(e^t)e^t = \frac{e^t\cos(e^t)}{2\sqrt{\sin(e^t)}}$$

Explanations

In this example when the chain rule is used to differentiate the square root expression, it must be used twice in succession. That's because this expression is the composite of a composite. The chain rule says that the derivative of f(g(h(t))) is:

$$\frac{d}{dt}f(g(h(t))) = f'(g(h(t)))\cdot g'(h(t))\cdot h'(t)$$

In this example:

$$f(t) = \sqrt{t}, g(t) = \sin(t) \text{ and } h(t) = e^t$$

In each mistake this compound chain rule computation is not performed correctly. The first mistake omits the g'(h(t)) factor. The second mistake has the form f'(g'(t))h'(t). The third mistake omits the h'(t) factor. The fourth mistake compounds the mistake made in the second attempt by placing the coefficient 1/2 *inside* the radical expression instead of the outside. *When repeated uses of the chain rule are required, take it one step at a time.*