Common Calculus Mistakes
Chain Rule: Inverse Sine

Some problems provide the opportunity for more than one mistake.

The Goal

Find

$$\frac{d}{dx}(\sin^{-1}(x^3))$$

The Mistakes

Find the mistakes:

1.

$$\frac{d}{dx}(\sin^{-1}(x^3)) = \frac{1}{\cos(x^3)(3x)}$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dx}(\sin^{-1}(x^3)) = \frac{1}{\cos(x^3)(3x)}$$

2.

$$\frac{d}{dx}(\sin^{-1}(x^3)) = -\sin^{-2}(x^3)\cos(x^3)3x^2$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dx}(\sin^{-1}(x^3)) = -\sin^{-2}(x^3)\cos(x^3)3x^2$$

3.

$$\frac{d}{dx}(\sin^{-1}(x^3)) = \frac{1}{\sqrt{1+x^2}}3x^2$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dx}(\sin^{-1}(x^3)) = \frac{1}{\sqrt{1+x^2}}3x^2$$

4.

$$\frac{d}{dx}(\sin^{-1}(x^3)) = \frac{1}{\sqrt{1-x^6}}\frac{1}{\sqrt{1-x^2}}3x^2$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dx}(\sin^{-1}(x^3)) = \frac{1}{\sqrt{1-x^6}}\frac{1}{\sqrt{1-x^2}}3x^2$$

5.

$$\frac{d}{dx}(\sin^{-1}(x^3)) = \frac{1}{\sqrt{1-x^6}}3x$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dx}(\sin^{-1}(x^3)) = \frac{1}{\sqrt{1-x^6}}3x$$

A Correct Solution

$$\frac{d}{dx}(\sin^{-1}(x^3)) = \frac{1}{\sqrt{1-x^6}}3x^2$$

(Roll the mouse over the area above to see the corrections in blue)

Explanations

In the first two mistakes the notation $\sin^{-1}(x)$ has been misunderstood to mean $1/\sin(x)$. That interpretation might seem to be a natural extension of notation such as $\sin^2(x)$, which does mean $(\sin(x))^2$, and $\sin^{-2}(x)$, which does mean $1/(\sin(x))^2$. However, for any function f(x), *by convention* when we write $f^{-1}(x)$, we mean instead the *inverse function* for f(x). So $\sin^{-1}(x)$ means *the inverse sine of x*, that is, the function that *undoes* the sine function. It is *not* equal to $1/\sin(x)$. Even under this false interpretation of $\sin^{-1}(x)$, in the first mistake the power rule is used incorrectly and the derivative of x^3 is not computed correctly.

In the third mistake the wrong formula for the derivative of $\sin^{-1}(x)$ was used, and the chain rule was used incorrectly, since the inner function x^3 should have been substituted for x in the proposed derivative of $\sin^{-1}(x)$. *Learn the correct derivative formulas* (visit Calculus

Facts for help in learning the rules).

The fourth mistake includes an extra factor that should not be there; so the chain rule was not used correctly. Finally, in the fourth mistake, the derivative of x^3 is incorrect. *Take care with even the "easy" parts of a problem*.