Common Calculus Mistakes
Quotient Rule

Some problems provide the opportunity for more than one mistake.

The Goal

Find

$$\frac{d}{dz}\left(\frac{5z^2}{4z^2-7}\right)$$

The Mistakes

Find the mistakes:

1.

$$\frac{d}{dz}\left(\frac{5z^2}{4z^2-7}\right) = \frac{10z-8z}{(4z^2-7)^2} = \frac{2z}{(4z^2-7)^2}$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dz}\left(\frac{5z^2}{4z^2-7}\right) = \frac{10z-8z}{(4z^2-7)^2} = \frac{2z}{(4z^2-7)^2}$$

2.

$$\frac{d}{dz}\left(\frac{5z^2}{4z^2-7}\right) = \frac{10z\cdot 4z^2-7 - (5z^2\cdot 8z)}{(4z^2-7)^2} = \frac{40z^3-7-40z^3}{(4z^2-7)^2} = \frac{-7}{(4z^2-7)^2}$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dz}\left(\frac{5z^2}{4z^2-7}\right) = \frac{10z\cdot (4z^2-7) - (5z^2\cdot 8z)}{(4z^2-7)^2} = \frac{40z^3-7-40z^3}{(4z^2-7)^2} = \frac{-7}{(4z^2-7)^2}$$

A Correct Solution

$$\frac{d}{dz}\left(\frac{5z^2}{4z^2-7}\right) = \frac{10z\cdot (4z^2-7) - (5z^2\cdot 8z)}{(4z^2-7)^2} = \frac{40z^3-70z-40z^3}{(4z^2-7)^2} = \frac{-70z}{(4z^2-7)^2}$$

Explanations

In the first mistake the quotient rule was not used correctly. The quotient rule is:

$$\frac{d}{dz}\left(\frac{f(z)}{g(z)}\right) = \frac{f'(z)g(z) - f(z)g'(z)}{(g(z))^2}$$

In the mistake the numerator was taken to be just f'(z) - g'(z). Oops!

In the second mistake parentheses were not used around the g(z) part of the expression in the numerator, with the result that only the first part of g(z) was multiplied by f'(z). *Use parentheses!*