Common Calculus Mistakes
Quotient Rule

Some problems provide the opportunity for more than one mistake.

The Goal

Find

$$\frac{d}{dt}\left(\frac{t-\ln t}{t^2}\right)$$

The Mistakes

Find the mistakes:

1.

$$\frac{d}{dt}\left(\frac{t-\ln t}{t^2}\right)=\frac{t^2\left(1-\frac{1}{\ln t}\right)-(t-\ln t)(2t)}{t^4}=\frac{t^2-\frac{t^2}{\ln t}-2t^2+2t\ln t}{t^4}=\frac{2t\ln t-t^2-\frac{t^2}{\ln t}}{t^4}$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dt}\left(\frac{t-\ln t}{t^2}\right)=\frac{t^2\left(1-\frac{1}{\ln t}\right)-(t-\ln t)(2t)}{t^4}=\frac{t^2-\frac{t^2}{\ln t}-2t^2+2t\ln t}{t^4}=\frac{2t\ln t-t^2-\frac{t^2}{\ln t}}{t^4}$$

2.

$$\frac{d}{dt}\left(\frac{t-\ln t}{t^2}\right)=\frac{\frac{1}{t}\cdot t^2-\ t-\ln t\ \cdot 2t}{(t^2)^2}=\frac{t-t-2t\ln t}{t^4}=\frac{-2t\ln t}{t^4}=\frac{-2\ln t}{t^3}$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dt}\left(\frac{t-\ln t}{t^2}\right)=\frac{?\frac{1}{t}\cdot t^2-?t-\ln t?\cdot 2t}{(t^2)^2}=\frac{t-t-2t\ln t}{t^4}=\frac{-2t\ln t}{t^4}=\frac{-2\ln t}{t^3}$$

3.

$$\frac{d}{dt}\left(\frac{t-\ln t}{t^2}\right)=\frac{1-\frac{1}{t}\ \cdot t^2-\ t-\ln t\ \cdot 2t}{(t^2)^2}=\frac{t^2-t-t-2t\ln t}{t^4}$$

$$=\frac{t^2-2t-2t\ln t}{t^4}=\frac{t-1-2\ln t}{t^3}$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dt}\left(\frac{t-\ln t}{t^2}\right)=\frac{?1-\frac{1}{t}?\cdot t^2-?t-\ln t?\cdot 2t}{(t^2)^2}=\frac{t^2-t-t-2t\ln t}{t^4}$$

$$=\frac{t^2-2t-2t\ln t}{t^4}=\frac{t-1-2\ln t}{t^3}$$

4.

$$\frac{d}{dt}\left(\frac{t-\ln t}{t^2}\right)=\frac{\left(1-\frac{1}{t}\right)t^{-2}-(t-\ln t)(-2t^{-3})}{(t^2)^2}=\frac{t^{-2}-t^{-3}+2t^{-2}-2t^{-3}\ln t}{t^4}$$

$$=\frac{3t^{-2}-t^{-3}-2t^{-3}\ln t}{t^4}=\frac{3t-1-2\ln t}{t^7}$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dt}\left(\frac{t-\ln t}{t^2}\right)=\frac{\left(1-\frac{1}{t}\right)t^{-2}-(t-\ln t)(-2t^{-3})}{(t^2)^2}=\frac{t^{-2}-t^{-3}+2t^{-2}-2t^{-3}\ln t}{t^4}$$

$$=\frac{3t^{-2}-t^{-3}-2t^{-3}\ln t}{t^4}=\frac{3t-1-2\ln t}{t^7}$$

5.

$$\frac{d}{dt}\left(\frac{t-\ln t}{t^2}\right)=\frac{2t(t-\ln t)-(t^2(1-\frac{1}{t}))}{(t^2)^2}=\frac{2t^2-2t\ln t-t^2+t}{t^4}$$

$$=\frac{t(t-2\ln t+1)}{t^4}=\frac{t-2\ln t+1}{t^3}$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dt}\left(\frac{t-\ln t}{t^2}\right)=\frac{2t(t-\ln t)-(t^2(1-\frac{1}{t}))}{(t^2)^2}=\frac{2t^2-2t\ln t-t^2+t}{t^4}$$

$$=\frac{t(t-2\ln t+1)}{t^4}=\frac{t-2\ln t+1}{t^3}$$

A Correct Solution

$$\frac{d}{dt}\left(\frac{t-\ln t}{t^2}\right) = \frac{(1-\frac{1}{t})t^2-(t-\ln t)2t}{(t^2)^2} = \frac{t^2-t-2t^2+2t\ln t}{t^4}$$

$$= \frac{t(2\ln t-t-1)}{t^4} = \frac{2\ln t-t-1}{t^3}$$

Explanations

In the first mistake the derivative of ln(t) was incorrectly computed to be 1/ln(t).

The second mistake has two mistakes. The derivative of the numerator was computed incorrectly - namely, the derivative of t is 1, not 0. Parentheses should have been used around the factor (t - ln(t)), since both t and ln(t) are part of the numerator of the original expression, and *both* need to be multiplied by 2t.

Parentheses were omitted in two places in the numerator in the third mistake, although the student continued on by assuming that the first set of parentheses were actually there. *Be very careful about using parentheses!*

The quotient rule is:

$$\frac{d}{dt}\left(\frac{f(t)}{g(t)}\right) = \frac{f'(t)g(t)-f(t)g'(t)}{(g(t))^2}$$

In the fourth mistake the student made the mistake of taking g(t) to be t^{-2} instead of the correct t^2.

In the fifth mistake the numerator terms were reversed in trying to use the quotient rule; this mistake changes the sign of the result. *Take care to get the order of the terms correct in the numerator when using the quotient rule.*