
Common Calculus Mistakes

Inverse Trigonometric Functions: Inverse Tangent

The Goal

Find

$$\frac{d}{dx}(\tan^{-1}(x^2))$$

The Mistake

Find the mistake:

$$\frac{d}{dx}(\tan^{-1}(x^2)) = \frac{\sqrt{1-x^4}}{\sqrt{1+x^4}}$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dx}(\tan^{-1}(x^2)) = \frac{\sqrt{1-x^4}}{\sqrt{1+x^4}}$$

The Correction

$$\frac{d}{dx}(\tan^{-1}(x^2)) = \frac{1}{1+(x^2)^2} \cdot 2x = \frac{2x}{1+x^4}$$

An Explanation

The student has not used the correct formula to find the derivative of inverse tangent, and there's no sign that the chain rule was used at all (we would see a $2x$ factor somewhere). The correct derivative formula for the inverse tangent function is:

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$