
Common Calculus Mistakes

Example: Tangent Lines

The Goal

Find the equation of the tangent line to

$$f(x) = x^3$$

at $x = 2$.

The Mistake

Find the mistake:

$$f(x) = x^3 \implies f'(x) = 3x^2.$$

Since also $f(2) = 8$, the tangent line is $y - 8 = 3x^2(x - 2)$, or $y = 3x^3 - 6x^2 + 8$.

Need a hint? Look carefully at the red part:

$$f(x) = x^3 \implies f'(x) = 3x^2.$$

Since also $f(2) = 8$, the tangent line is $y - 8 = 3x^2(x - 2)$, or $y = 3x^3 - 6x^2 + 8$.

The Correction

$$f(x) = x^3 \implies f'(x) = 3x^2.$$

Therefore the slope of the tangent line at $x = 2$ is $f'(2) = 12$.

Since also $f(2) = 8$, the tangent line is $y - 8 = 12(x - 2)$, or $y = 12x - 16$.

An Explanation

The derivative of a function $f(x)$ provides a *formula* $f'(x)$ from which the slope of a tangent line can be computed at a point $x=a$ by *evaluating* $f'(x)$ at $x=a$, that is, by computing $m=f'(a)$. The result is a *number*, which then can be used to find the equation of the tangent line at $x=a$ using the point-slope form for a line, in this situation: $y - f(a) = m(x-a)$.

(Note that in the mistaken solution the proposed tangent line equation is *not* the equation of a *line*.)