Common Calculus Mistakes
Chain Rule: Natural Logarithm

The Goal

Find

$$\frac{d}{dS}\left(k\ln\left(\frac{S}{S_0}\right)\right),$$

where k and S_0 are constants.

The Mistake

Find the mistake:

$$\frac{d}{dS}\left(k\ln\left(\frac{S}{S_0}\right)\right) = k\frac{1}{S\cdot S_0}\left(\frac{S_0}{S_0^2}\right)$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dS}\left(k\ln\left(\frac{S}{S_0}\right)\right) = k\frac{1}{S\cdot S_0}\left(\frac{S_0}{S_0^2}\right)$$

The Correction

$$\frac{d}{dS}\left(k\ln\left(\frac{S}{S_0}\right)\right) = k\frac{1}{\frac{S}{S_0}}\left(\frac{1}{S_0}\right) = \frac{k}{S}$$

An Explanation

The *chain rule* is applied to the natural logarithm of a function g(x) as follows:

$$\frac{d}{dx}(f(g(x)) = f'(g(x))g'(x) \implies \frac{d}{dx}(\ln(g(x)) = \frac{1}{g(x)}g'(x)$$

In this example $g(x) = S/S_0$, so the first part of the derivative should be $1/(S/S_0)$ and this should be multiplied by $g'(x) = 1/S_0$ (think of S/S_0 as the constant $1/S_0$ times S).

(In the mistake the student did successfully find g'(x), apparently using the *quotient rule* to get S_0/S_0^2, which makes the derivative harder than necessary. For this reason that part of the mistake is also marked in red.)