Common Calculus Mistakes
Power Rule (with parameters)

Some problems provide the opportunity for more than one mistake.

The Goal

Find

$$\frac{d}{dx}\left(\frac{1}{2}wLx-\frac{1}{2}wx^2\right),$$

where w and L are constants.

The Mistakes

Find the mistakes:

1.

$$\frac{d}{dx}\left(\frac{1}{2}wLx-\frac{1}{2}wx^2\right)=\frac{1}{2}wL-2wx$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dx}\left(\frac{1}{2}wLx-\frac{1}{2}wx^2\right)=\frac{1}{2}wL-\color{red}{2wx}$$

2.

$$\frac{d}{dx}\left(\frac{1}{2}wLx-\frac{1}{2}wx^2\right)=\frac{1}{2}(Lw)+\frac{w}{2}L-\left(\frac{1}{2}x^2+\frac{w}{2}(2x)\right)$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dx}\left(\frac{1}{2}wLx-\frac{1}{2}wx^2\right)=\color{red}{\frac{1}{2}(Lw)+\frac{w}{2}L-\left(\frac{1}{2}x^2+\frac{w}{2}(2x)\right)}$$

3.

$$\frac{d}{dx}\left(\frac{1}{2}wLx-\frac{1}{2}wx^2\right)=\frac{d}{dx}\left((2wLx)^{-1}-2(wx^2)^{-1}\right)=-(2wLx)^{-2}2wL+2(wx^2)^{-2}2wx$$

Need a hint? Look carefully at the red part:

$$\frac{d}{dx}\left(\frac{1}{2}wLx-\frac{1}{2}wx^2\right)=\frac{d}{dx}\left((2wLx)^{-1}-2(wx^2)^{-1}\right)=-(2wLx)^{-2}2wL+2(wx^2)^{-2}2wx$$

A Correct Solution

$$\frac{d}{dx}\left(\frac{1}{2}wLx-\frac{1}{2}wx^2\right)=\frac{1}{2}wL-wx$$

Explanations

The presence of *parameters*, or arbitrary constants, like w and L, seem to give students all kinds of trouble in calculus (or any prior class). The correct approach is to treat these constants as *numbers* whose values happen not to be specified. The constant multiple rule should then be used to pull these constants out in front of any expression that is to be differentiated.

The first mistake is just one of forgetting to multiply the 2 arising from differentiating x^2 by the constant 1/2.

In the second mistake the student has attempted (incorrectly) to apply the product rule to differentiate each of the two terms; the constant L has been wrongly treated as a variable (in particular, as being equivalent to x). Moreover, even taking L as a variable in this way, the product rule was used incorrectly on the first term.

In the third mistake it appears that the student misinterpreted the meaning of the fraction notation by assuming that all of the parameters and x factors appear in the respective denominators (although the "2" in the second term was for some reason placed in the numerator).

The correct solution shows how the constant multiple rule boils the problem down to finding the derivatives of x and x^2.