Common Calculus Mistakes
Implicit Differentiation: Multiple Mistakes

Some problems provide the opportunity for more than one mistake.

The Goal

Find dy/dx, where x and y are related by the equation

$$\ln(xy) = 2x$$

The Mistakes

Find the mistakes:

1.

$$\ln(xy) = 2x \implies \frac{1}{y + x\frac{dy}{dx}} = 2 \implies y + x\frac{dy}{dx} = 2 \implies x\frac{dy}{dx} = 2 - y \implies \frac{dy}{dx} = \frac{2-y}{x}$$

Need a hint? Look carefully at the red part:

$$\ln(xy) = 2x \implies \frac{1}{y + x\frac{dy}{dx}} = 2 \implies y + x\frac{dy}{dx} = 2 \implies x\frac{dy}{dx} = 2 - y \implies \frac{dy}{dx} = \frac{2-y}{x}$$

2.

$$\ln(xy) = 2x \implies \ln(x) + \ln(y) = 2x \implies \frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 2 \implies \frac{1}{y}\frac{dy}{dx} = 2x \implies \frac{dy}{dx} = 2xy$$

Need a hint? Look carefully at the red part:

$$\ln(xy) = 2x \implies \ln(x) + \ln(y) = 2x \implies \frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 2 \implies \frac{1}{y}\frac{dy}{dx} = 2x \implies \frac{dy}{dx} = 2xy$$

A Correct Solution

$$\ln(xy) = 2x \implies \frac{1}{xy}\left(y + x\frac{dy}{dx}\right) = 2 \implies y + x\frac{dy}{dx} = 2xy$$
$$\implies x\frac{dy}{dx} = 2xy - y \implies \frac{dy}{dx} = \frac{2xy - y}{x}$$

Explanations

In the first mistaken solution the derivative of ln(xy) with respect to x was computed incorrectly. By the *chain rule* the derivative of ln(Q) is 1/Q times the derivative of Q; in this case, 1/(xy) times the derivative of xy with respect to x. The mistaken solution incorrectly has the derivative of ln(Q) as 1/Q'. In the next step there is a second mistake - the reciprocal of the left side is taken, but the right side is left unchanged.
In the second mistaken solution the student started by using a logarithm rule to simplify the ln(xy) term. This made it easier to find the derivative. Alas, in the step following the differentiation, 1/x was *subtracted* from the left side but the right side was *multiplied* by x.