Common Algebra Mistakes
Example: Solving a Quadratic Equation

The Goal

Find all pairs (x,y) (for x and y both real numbers) satisfying the equation:

$$x^2 + 4y^2 = 0$$

The Mistake

Find the algebra mistake:

$$x^2 + 4y^2 = 0 \implies 4y^2 = -x^2 \implies y^2 = -\frac{1}{4}x^2 \implies y = \pm\sqrt{-\frac{x^2}{4}}$$

Need a hint? Look carefully at the red part of the algebra:

$$x^2 + 4y^2 = 0 \implies 4y^2 = -x^2 \implies y^2 = -\frac{1}{4}x^2 \implies y = \pm\sqrt{-\frac{x^2}{4}} \implies ?$$

The Correction

$$x^2 + 4y^2 = 0 \implies x = 0 \text{ and } y = 0 \text{ since } x^2 + 4y^2 > 0 \text{ otherwise}$$

An Explanation

There is no mistake in the algebra, only a failure to notice at the start that $x^2 + 4y^2$ can only be zero if both x and y are zero.

(Once the algebra had been done, note that the quantity under the square root can only have a real value if x=0, and therefore also y=0.)