Common Algebra Mistakes

Example: Radical Expressions - Distance Formula

The Goal

Find the distance D between the points (x,0,0) and (x,y,z) in 3-space.

The Mistake

Find the algebra mistake:

$$D = \sqrt{(x-x)^2 + (y-0)^2 + (z-0)^2} = \sqrt{0 + y^2 + z^2} = \sqrt{y^2 + z^2} = y + z$$

Need a hint? Look carefully at the red part of the algebra:

$$D = \sqrt{(x-x)^2 + (y-0)^2 + (z-0)^2} = \sqrt{0 + y^2 + z^2} = \sqrt{y^2 + z^2} = y + z$$

The Correction

$$D = \sqrt{(x-x)^2 + (y-0)^2 + (z-0)^2} = \sqrt{0 + y^2 + z^2} = \sqrt{y^2 + z^2} = \sqrt{y^2 + z^2}$$

An Explanation

The mistake is in the final step - it is incorrect to distribute the square root over the sum. That is:

$$\sqrt{A+B} \neq \sqrt{A} + \sqrt{B}$$

Try it with some numbers to convince yourself: let A=1 and B=1. The left side is √2, while the right side is 2. One *counterexample* disproves a general formula.