Common Algebra Mistakes
Example: Solving a Logarithm Equation

The Goal

Solve the following equation for x (where a and b are (positive) constants):

$$a\ln(bx)+a=0$$

The Mistake

Find the algebra mistake:

$$a\ln(bx)+a=0 \implies a\ln(bx)=a \implies \ln(bx)=1 \implies x=b^{-1}$$

Need a hint? Look carefully at the red part of the algebra:

$$a\ln(bx)+a=0 \implies a\ln(bx)=a \implies \ln(bx)=1 \implies x=b^{-1}$$

The Correction

$$a\ln(bx)+a=0 \implies a\ln(bx)=-a \implies \ln(bx)=-1$$
$$\implies bx=e^{-1} \implies x=\frac{e^{-1}}{b}=\frac{1}{be}$$

An Explanation

There are two mistakes. The first is a careless error; a must be *subtracted* from both sides to isolate the a·ln(bx) term. The second mistake apparently was to assume that ln(1) = 1 and hence bx = 1. But ln(1) = 0. The correct way to proceed is to use the equivalence

$$\ln(A)=B \iff A=e^{B},$$

which we can also view as an action by saying we take e to the expression on each side, and since $y=e^x$ and y=ln(x) are inverse functions of each other, the left side simplifies to the inside of the natural logarithm expression.