Common Algebra Mistakes

Example: Solving a Logarithm Equation

The Goal

Solve the following equation for x:

$$\log_4 x + \log_4 (x-3) = 1$$

The Mistake

Find the algebra mistake:

$$\log_4 x + \log_4 (x-3) = 1 \implies \log_4 (x(x-3)) = \log_4 4 \implies x(x-3) = 4$$
$$\implies x^2 - 3x = 4 \implies x^2 - 3x - 4 = 0 \implies (x-4)(x+1) = 0 \implies x = -1 \text{ or } x = 4$$

Need a hint? Look carefully at the red part of the algebra:

$$\log_4 x + \log_4 (x-3) = 1 \implies \log_4 (x(x-3)) = \log_4 4 \implies x(x-3) = 4$$
$$\implies x^2 - 3x = 4 \implies x^2 - 3x - 4 = 0 \implies (x-4)(x+1) = 0 \implies x = -1 \text{ or } x = 4$$

The Correction

$$\log_4 x + \log_4 (x-3) = 1 \implies \log_4 (x(x-3)) = \log_4 4 \implies x(x-3) = 4$$
$$\implies x^2 - 3x = 4 \implies x^2 - 3x - 4 = 0 \implies (x-4)(x+1) = 0 \implies x = -1 \text{ or } x = 4$$

However, $x = -1$ is not a solution since $\log_4(-1)$ is not defined. Thus $x = 4$.

An Explanation

Check that the computed solutions to an equation actually *are* solutions; in this case one of the two proposed solutions does not lie in the domain of the logarithm function. We call such "solutions" extraneous.

The problem arises because the implications in the solution process are not always reversible, that is, are not always equivalences. So it's certainly true that a solution x of the initial equation is a solution of the equation in the second step. However, the solution x=-1 of the equation in the second step is not a solution of the initial equation.