Common Algebra Mistakes
Example: Radical Expression

The Goal

Simplify the expression:

$$\frac{\sqrt{x-1+h}-\sqrt{x-1}}{h}$$

The Mistake

Find the algebra mistake:

$$\frac{\sqrt{x-1+h}-\sqrt{x-1}}{h}=\frac{(\sqrt{x}-1+h)-(\sqrt{x}-1)}{h}=\frac{\sqrt{x}-1+h-\sqrt{x}+1}{h}=\frac{h}{h}=1$$

Need a hint? Look carefully at the red part of the algebra:

$$\frac{\sqrt{x-1+h}-\sqrt{x-1}}{h}=\frac{(\sqrt{x}-1+h)-(\sqrt{x}-1)}{h}=\frac{\sqrt{x}-1+h-\sqrt{x}+1}{h}=\frac{h}{h}=1$$

The Correction

$$\frac{\sqrt{x-1+h}-\sqrt{x-1}}{h}=\frac{\sqrt{x-1+h}-\sqrt{x-1}}{h}\cdot\frac{\sqrt{x-1+h}+\sqrt{x-1}}{\sqrt{x-1+h}+\sqrt{x-1}}$$

$$=\frac{(x-1+h)-(x-1)}{h(\sqrt{x-1+h}+\sqrt{x-1})}=\frac{x-1+h-x+1}{h(\sqrt{x-1+h}+\sqrt{x-1})}$$

$$=\frac{h}{h(\sqrt{x-1+h}+\sqrt{x-1})}=\frac{1}{\sqrt{x-1+h}+\sqrt{x-1}}$$

An Explanation

Terms *cannot* simply be pulled out of a square root expression. This expression cannot be simplified as shown in "The Mistake".

In calculus it is useful to "simplify" expressions (such as this one) as shown in "The Correction". The technique used here eliminates square roots from the numerator at the expense of introducing them in the denominator. You may have been told that square roots in the denominator should be removed by "rationalizing". While that is an important skill,

sometimes it is beneficial to introduce or keep square root expressions in the denominator. In the expression here it allows both the numerator and denominator to be reduced to a form in which the *limit* can be computed as h appoaches 0.