Common Algebra Mistakes
Example: Simplifying an Expression

The Goal

Simplify the expression:

$$\frac{\sqrt{x}\,\frac{1}{x} - \ln(x)\frac{1}{2}x^{-\frac{1}{2}}}{x}$$

The Mistake

Find the algebra mistake:

$$\frac{\sqrt{x}\,\frac{1}{x} - \ln(x)\frac{1}{2}x^{-\frac{1}{2}}}{x} = \frac{\sqrt{x} - \ln(x)}{2x^2\sqrt{x}}$$

Need a hint? Look carefully at the red part of the algebra:

$$\frac{\sqrt{x}\,\frac{1}{x} - \ln(x)\frac{1}{2}x^{-\frac{1}{2}}}{x} = \frac{\sqrt{x} - \ln(x)}{\color{red}{2x^2\sqrt{x}}}$$

The Correction

$$\frac{\sqrt{x}\,\frac{1}{x} - \ln(x)\frac{1}{2}x^{-\frac{1}{2}}}{x} = \frac{x^{-\frac{1}{2}} - \ln(x)\frac{1}{2}x^{-\frac{1}{2}}}{x}\cdot\frac{2\sqrt{x}}{2\sqrt{x}} = \frac{x^{-\frac{1}{2}}2\sqrt{x} - \ln(x)\frac{1}{2}x^{-\frac{1}{2}}2\sqrt{x}}{x2\sqrt{x}} = \frac{2 - \ln(x)}{2x^{\frac{3}{2}}}$$

An Explanation

If we look carefully at the mistake, we see that the expression has the form

$$\frac{a\frac{1}{b} - c\frac{1}{d}}{e} \quad \text{where } a = \sqrt{x}, b = x, c = \ln(x), d = 2\sqrt{x} \text{ and } e = x,$$

which the student has mistakenly rewritten in the form

$$\frac{a - c}{ebd}.$$

But only *common factors* of the numerator may be rewritten this way, for example, if the

expression had the form

$$\frac{a^{\frac{1}{b}} - c^{\frac{1}{b}}}{e} = \frac{(a-c)^{\frac{1}{b}}}{e},$$

then it could be rewritten as follows:

$$\frac{a^{\frac{1}{b}} - c^{\frac{1}{b}}}{e} = \frac{a-c}{eb}.$$

A nice way to simplify the original expression is to multiply by 1 in the form shown in "The Correction", chosen so that the negative exponent factors in the numerator are cleared.