Common Algebra Mistakes
Example: Solving an Exponential Equation

The Goal

Solve the following equation for t:

$$20 = 10e^{0.013t}$$

The Mistake

Find the algebra mistake:

$$20 = 10e^{0.013t} \implies \ln(20) = \ln(10)(0.013t)$$
$$\implies 0.013t = \frac{\ln(20)}{\ln(10)} \implies \frac{\ln(20)}{0.013\ln(10)}$$

Need a hint? Look carefully at the red part of the algebra:

$$20 = 10e^{0.013t} \implies \ln(20) = \ln(10)(0.013t)$$
$$\implies 0.013t = \frac{\ln(20)}{\ln(10)} \implies \frac{\ln(20)}{0.013\ln(10)}$$

The Correction

$$20 = 10e^{0.013t} \implies \ln(20) = \ln(10) + (0.013t)$$
$$\implies 0.013t = \ln(20) - \ln(10) \implies \frac{\ln(20) - \ln(10)}{0.013}$$

An Explanation

The rules of logarithms were incorrectly used. The log of a product is the *sum* of the logs of the factors.

For this equation a more efficient solution would be to start by dividing both sides by 10:

$$20 = 10e^{0.013t} \implies e^{0.013t} = 2 \implies 0.013t = \ln(2) \implies t = \frac{\ln(2)}{0.013}$$