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# Common Algebra Mistakes

## Example: Solving a Quadratic Equation

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### The Goal

Solve the following equation for  $c$ :

$$c^2 + 7c = 6$$

### The Mistake

Find the algebra mistake:

$$c^2 + 7c = 6 \implies c(c+7) = 6 \implies c = 6$$

Need a hint? Look carefully at the red part of the algebra:

$$c^2 + 7c = 6 \implies c(c+7) = 6 \implies c = 6$$

### The Correction

$$\begin{aligned} c^2 + 7c = 6 &\implies c^2 + 7c - 6 = 0 \\ \implies c &= \frac{-7 \pm \sqrt{7^2 - 4(1)(-6)}}{2(1)} = \frac{-7 \pm \sqrt{49 + 24}}{2} = \frac{-7 \pm \sqrt{73}}{2} \end{aligned}$$

### An Explanation

(In this explanation we cannot use  $c$  for the constant coefficient of a general quadratic since the variable in the quadratic is  $c$ . We use  $d$  instead.)

It is no help in solving a quadratic equation to arrange it in the form  $ac^2 + bc = d$  (unless  $d = 0$ ) (in this problem,  $c^2 + 7c = 6$ ) and then write it in the form  $c(ac + b) = d$  (in this problem,  $c(c + 7) = 6$ ). If  $d$  is not 0, you *cannot* conclude that  $c = d$  (or  $ac + b = d$ ) (in this case  $c = 6$  (or  $c + 7 = 6$ )). You can check that neither 6 nor -1 are solutions of  $c^2 + 7c = 6$ .

The number 0 is the *only* number with the property that if a product is equal to that number, then one of the factors is that number. If we had the equation  $c(c + 7) = 0$  it would be true that  $c = 0$  or  $c + 7 = 0$ .

There are *three* basic ways to solve a quadratic equation:

- (1) factor (the equation  $c^2 + 7c - 6 = 0$  does not easily factor, so use a different method)
- (2) complete the square:

$$c^2 + 7c = 6 \implies c^2 + 7c + \left(\frac{7}{2}\right)^2 = 6 + \left(\frac{7}{2}\right)^2 = 6 + \frac{49}{4} = \frac{24 + 49}{4} = \frac{73}{4}$$

$$\implies \left(c + \frac{7}{2}\right)^2 = \frac{73}{4} \implies c + \frac{7}{2} = \pm \sqrt{\frac{73}{4}} = \pm \frac{\sqrt{73}}{2}$$

$$\implies c = -\frac{7}{2} \pm \frac{\sqrt{73}}{2} = \frac{-7 \pm \sqrt{73}}{2}$$

(3) use the quadratic formula: the solutions of  $ac^2 + bc + d = 0$  are

$$c = \frac{-b \pm \sqrt{b^2 - 4ad}}{2a}$$

(see The Correction for the solution to the equation  $c^2 + 7c - 6 = 0$  using the quadratic formula)