Common Algebra Mistakes
Example: Solving a Quadratic Equation

The Goal

Solve the following equation for t:

$$t^2 - 6t - 3 = 0$$

The Mistake

Find the algebra mistake:

$$t^2 - 6t - 3 = 0 \implies t^2 - 6t = 3 \implies t(t-6) = 3$$
$$\implies t = 0 \text{ or } t - 6 = 3 \implies t = 0 \text{ or } t = 9$$

Need a hint? Look carefully at the red part of the algebra:

$$t^2 - 6t - 3 = 0 \implies t^2 - 6t = 3 \implies t(t-6) = 3$$
$$\implies t = 0 \text{ or } t - 6 = 3 \implies t = 0 \text{ or } t = 9$$

The Correction

$$t^2 - 6t - 3 = 0 \implies t = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-3)}}{2} = \frac{6 \pm \sqrt{36+12}}{2} = \frac{6 \pm \sqrt{48}}{2}$$
$$\implies t = \frac{6 \pm \sqrt{4}\sqrt{12}}{2} = \frac{6 \pm 2\sqrt{12}}{2} = 3 \pm \sqrt{12}$$

An Explanation

It is no help in solving a quadratic equation to arrange it in the form $at^2 + bt = c$ (unless c = 0) (in this problem, $t^2 - 6t = 3$) and then write it in the form t(at + b) = c (in this problem, t(t - 6) = 3). If c is not 0, you *cannot* conclude that t = c or at + b = c (in this case t = 3 or t - 6 = 3). In this problem you can check that neither 3 nor 9 are solutions of the original equation.
The number 0 is the *only* number with the property that if a product is equal to that number, then one of the factors is that number. If we had the equation t(t - 6) = 0 it would be true that t = 0 or t - 6 = 0.
There are *three* basic ways to solve a quadratic equation:
(1) factor (the equation $t^2 - 6t - 3 = 0$ does not easily factor, so use a different method)
(2) complete the square:

$$t^2 - 6t - 3 = 0 \implies (t^2 - 6t + 9) - 3 = 9 + 0 \implies (t-3)^2 = 9 + 3$$
$$\implies (t-3)^2 = 12 \implies t - 3 = \pm\sqrt{12} \implies t = 3 \pm \sqrt{12}$$

(3) use the quadratic formula: the solutions of $at^2 + bt + c = 0$ are

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(see The Correction for the solution to the equation $t^2 - 6t - 3 = 0$ using the quadratic formula)
