Common Algebra Mistakes
Example: Function Notation

The Goal

Evaluate f(x+h) for the function:

$$f(x) = \sqrt{x^2 - 1}$$

The Mistake

Find the algebra mistake:

$$f(x) = \sqrt{x^2 - 1} \implies f(x+h) = \sqrt{x^2 - 1} + h$$

Need a hint? Look carefully at the red part of the algebra:

$$f(x) = \sqrt{x^2 - 1} \implies f(x+h) = \sqrt{x^2 - 1} + h$$

The Correction

$$f(x) = \sqrt{x^2 - 1} \implies f(x+h) = \sqrt{(x+h)^2 - 1}$$

An Explanation

There are *very few* functions f(x) for which the equation f(x+h) = f(x) + h is true (it is true if y = f(x) is a straight line with slope 1).
The mistake in thinking that f(x+h) = f(x) + h is true in general comes from not properly understanding function notation.
In f(x) the x is a *placeholder*. To evaluate f(x+h) we must put x+h in for that placeholder *precisely where x appears* in f(x) - no more and no less.
Follow the sequence of examples:

The function f(x):

$$f(x) = \sqrt{x^2 - 1}$$

Whatever we replace x with replaces x in the formula (even something silly like *):

$$f(*) = \sqrt{*^2 - 1}$$

Replace x with 5 to find f(5):

$$f(5) = \sqrt{5^2 - 1} = \sqrt{25 - 1} = \sqrt{24}$$

Note that 5 can be written as 4+1:

$$f(4+1) = \sqrt{(4+1)^2 - 1} = \sqrt{5^2 - 1} = \sqrt{25 - 1} = \sqrt{24}$$

Following the pattern we can find f(4+h):

$$f(4+h) = \sqrt{(4+h)^2 - 1} = \sqrt{(16+2h+h^2) - 1} = \sqrt{h^2+2h+15}$$

And f(x+h) is a matter of replacing the 4 by x:

$$f(x+h) = \sqrt{(x+h)^2 - 1} = \sqrt{(x^2+2xh+h^2) - 1} = \sqrt{x^2+2xh+h^2-1}$$