Common Algebra Mistakes
Example: Function Notation

The Goal

Evaluate f(x+h) for the function:

$$f(x) = \sqrt{2x+1}$$

The Mistake

Find the algebra mistake:

$$f(x) = \sqrt{2x+1} \implies f(x+h) = \sqrt{2x+1}+h$$

Need a hint? Look carefully at the red part of the algebra:

$$f(x) = \sqrt{2x+1} \implies f(x+h) = \sqrt{2x+1}+h$$

The Correction

$$f(x) = \sqrt{2x+1} \implies f(x+h) = \sqrt{2(x+h)+1}$$

An Explanation

There are *very few* functions f(x) for which the equation f(x+h) = f(x) + h is true (it is true if y = f(x) is a straight line with slope 1).
The mistake in thinking that f(x+h) = f(x) + h is true in general comes from not properly understanding function notation.
In f(x) the x is a *placeholder*. To evaluate f(x+h) we must put x+h in for that placeholder *precisely where x appears* in f(x) - no more and no less.
Follow the sequence of examples:

The function f(x):

$$f(x) = \sqrt{2x+1}$$

Whatever we replace x with replaces x in the formula (even something silly like @):

$$f(@) = \sqrt{2@+1}$$

Replace x with 4 to find f(4):

$$f(4) = \sqrt{2(4)+1} = \sqrt{8+1} = \sqrt{9} = 3$$

Note that 4 can be written as 3+1:

$$f(3+1) = \sqrt{2(3+1)+1} = \sqrt{2(4)+1} = \sqrt{8+1} = \sqrt{9} = 3$$

Following the pattern we can find f(3+h):

$$f(3+h) = \sqrt{2(3+h)+1} = \sqrt{6+2h+1} = \sqrt{7+2h}$$

And f(x+h) is a matter of replacing the 3 by x:

$$f(x+h) = \sqrt{2(x+h)+1} = \sqrt{2x+2h+1}$$